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New *n*-Way Hybrid Power Dividers

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Abstract—**A new *n*-way planar hybrid power divider (henceforth HPD) and an *n*-way coaxial-type HPD are proposed, and the synthesis methods of three-way and four-way planar HPD's are shown. As for the three-way planar HPD, the isolation characteristics among three ports show more than 20 dB, and the VSWR's of four ports show less than 1.4 in 2 : 1 bandwidth. The experimental characteristics are in good agreement with the theoretical analysis.**

I. INTRODUCTION

THE OBJECTIVE of this paper is to propose some new n-way hybrid power dividers (henceforth HPD) constructed by a new synthesis method. The n-way HPD's described by Wilkinson [1] and Yee *et al.* [4] are synthesized by using M sections of n uncoupled transmission lines of equal length with isolation resistors of the Y connection which are connected from the end of the n transmission lines to a common junction. The isolation resistors cannot be designed to be a planar structure. The n-way HPD's presented in this paper are synthesized by using M sections of n-wire coupled (or uncoupled) lines of equal length with isolation resistors which are connected by the ends of the neighboring wires.

The analysis of the n-way HPD is done by getting the eigenvalues and the corresponding eigenvectors of the characteristic admittance matrices of M sections for n-wire coupled (or uncoupled) lines and of M admittance matrices for the isolation resistors, and then by getting the equivalent circuit representation of the n-way HPD [8], [9]. The equivalent circuit representation is presented by n circuits which consist of a two-port for the even-mode circuit and $n - 1$ one-ports for the odd-mode circuits.

It can be shown that the $(n + 1)$-port made with a coupled n-wire line with isolation resistors of the Y-connection acts as an n-way HPD at narrow-band frequencies [6], [8], [9]. If we use isolation resistors different from the Y-connection, it needs some sections of isolation resistors and n-wire segments to perform matching and isolation among output ports at the required frequency.

This paper proposes an n-way HPD of a planar structure and a coaxial type n-way HPD, constructed by way of new connections of isolation resistors. As for the examples of the planar HPD, we show the circuits of three-way and four-way HPD's, and their VSWR's and isolation responses are shown in the figures. The isolation characteristics in three output ports of the three-way HPD show more than 20 dB, and the VSWR's of four ports show less than 1.4 in 2 : 1 bandwidth. The isolation characteristics in four ports of the four-way HPD show more than 24 dB, and the VSWR's of five ports show less than 1.2 in 2 : 1 bandwidth.

II. THE n-WAY PLANAR HYBRID POWER DIVIDER

Since we get isolation resistors of a planar structure by connecting $n - 1$ resistors by the ends of the neighboring wires of the n-wire lines constituting an n-way HPD, we consider a $(1,n)$-port, shown in Fig. 1(a), which is designed by using M sections of n uncoupled transmission lines of equal length with the planar isolation resistors. The analysis

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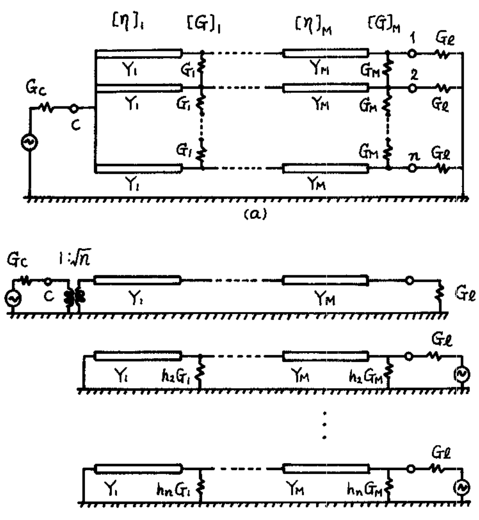

Fig. 1. (a) The n-way hybrid power divider of planar structure. (b) The equivalent circuit representation.

of the circuit can be done by getting the eigenvalues and the corresponding eigenvectors of the characteristic admittance matrices of the M sections for the n transmission lines and of the M admittance matrices for the planar isolation resistors. In reference to this circuit, we define the $n \times n$ real symmetric matrix $[H]$ as follows:

$$[H] = \begin{bmatrix} 1 & -1 & 0 & \cdots & \cdots & 0 \\ -1 & 2 & -1 & & & \vdots \\ 0 & \ddots & \ddots & \ddots & & 0 \\ \vdots & & -1 & 2 & -1 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \quad (1)$$

The eigenvalues h_i $(i = 1, 2, \cdots, n)$ of matrix $[H]$ are obtained as

$$h_i = 2 - 2 \cos \pi (i - 1)/n \quad (i = 1, 2, \cdots, n). \quad (2)$$

The eigenvector corresponding to the eigenvalue $h_1 = 0$ is $[1 \ 1 \cdots 1]^t$ (where $[M]^t$ presents the transpose of $[M]$). The eigenvectors corresponding to the eigenvalues h_i $(i = 2, \cdots, n)$ are

$$[\sin \phi_i \ \sin 2\phi_i - \sin \phi_i \cdots \sin k\phi_i - \sin (k-1)\phi_i \cdots \sin n\phi_i - \sin (n-1)\phi_i]^t \quad (3)$$

where $\phi_i = (i - 1)\pi/n$. By normalizing these eigenvectors, we get a real orthogonal matrix $[P_H]$.

The admittance matrices $[G]_\mu$ of the planar isolation resistors can be represented according to matrix $[H]$ as

$$[G]_\mu = G_\mu [H] \quad (\mu = 1, \cdots, M). \quad (4)$$

Therefore,

$$[P_H]^t [G]_\mu [P_H] = \text{diag}\,[0, G_\mu h_2, \cdots, G_\mu h_n]. \quad (5)$$

The characteristic admittance matrices $[\eta]_\mu$ of the n transmission lines are presented

$$[\eta]_\mu = Y_\mu 1_n \quad (\mu = 1, \cdots, M) \quad (6)$$

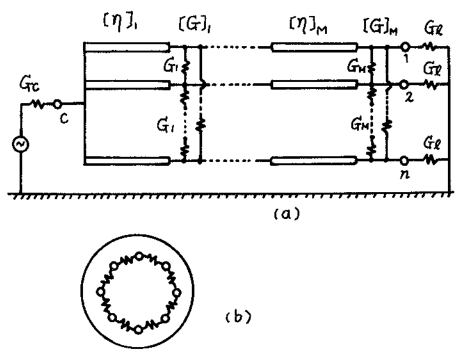

Fig. 2. (a) Coaxial-type n-way hybrid power divider. (b) The inner arrangement of n wires and resistors.

where 1_n represents the $n \times n$ identity matrix. Therefore,

$$[P_H]^t [\eta]_\mu [P_H] = Y_\mu 1_n \quad (\mu = 1, \cdots, M). \quad (7)$$

Since all the admittance matrices of the planar isolation resistors and all the characteristic admittance matrices of the transmission lines are transformed into diagonal matrices according to the orthogonal matrix $[P_H]$, the $(1,n)$-port shown in Fig. 1(a) can be equivalently transformed into a two-port of the even-mode circuit and $n - 1$ one-ports of the odd-mode circuits as shown in Fig. 1(b) [8], [9]. The two-port circuit is represented by a quarter-wave transformer of M sections [10]. The characteristic admittances $Y_1, \cdots, Y_M$ are decided by the transducer-loss characteristic of the maximally flat transformer or the Tchebyscheff transformer.

As for the $n - 1$ one-port circuits, if the following $n - 1$ equations are satisfied

$$G_l = h_i G_M + \cfrac{Y_M^2}{h_i G_{M-1} + \cfrac{Y_{M-1}^2}{\ddots \; h_i G_3 + \cfrac{Y_3^2}{h_i G_2 + \cfrac{Y_2^2}{h_i G_1}}}} \quad (i = 2, \cdots, n) \quad (8)$$

then each one-port matches at the center frequency where electrical length of the line section θ equals $\pi/2$. Since $Y_1, \cdots, Y_M$ have been selected, if $M = n - 1$, then $G_1, \cdots, G_M$ can be selected. It has been shown that, if the two-port circuit and all the $n - 1$ one-port circuits, which represent the equivalent circuit representation for the given $(n + 1)$-port, match at the center frequency, then the given $(n + 1)$-port acts as an n-way HPD [8], [9]. Therefore, the n-way HPD needs $n - 1$ sections of the n-wire lines and the planar isolation resistors for matching at the center frequency.

III. Coaxial Type n-Way Hybrid Power Divider

Since we get the coaxial-type isolation resistors by connecting n resistors by the ends of the neighboring wires of the n-wire lines constituting an n-way HPD, we consider a $(1,n)$-port, shown in Fig. 2, which is designed by using M sections of n-wire coaxial-type lines of equal length with the coaxial-type isolation resistors. In reference to this circuit, we define $n \times n$ real symmetric matrix $[F]$ as follows:

$$[F] = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 & -1 \\ -1 & 2 & -1 & \ddots & & 0 \\ 0 & -1 & 2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & & \ddots & \ddots & 2 & -1 \\ -1 & 0 & \cdots & 0 & -1 & 2 \end{bmatrix}. \tag{9}$$

The eigenvalues of matrix $[F]$ have some double roots and the number is

$$\left.\begin{aligned} &(n-2)/2, \quad \text{for } n \text{ even} \\ &(n-1)/2, \quad \text{for } n \text{ odd} \end{aligned}\right\}. \tag{10}$$

The eigenvalues f_i are obtained as

$$\left.\begin{aligned} f_1 &= 0 \\ f_{2i} \text{ and } f_{2i+1} &= 2 - 2\cos 2\pi i/n \end{aligned}\right\}$$
$$(i = 1, \cdots, n/2 \text{ or } i = 1, \cdots, (n-1)/2). \tag{11}$$

When we make a real orthogonal matrix $[P_F]$ according to the eigenvectors of matrix $[F]$, the following equation is obtained:

$$[P_F]^t[F][P_F] = \operatorname{diag}[0, f_2, \cdots, f_n]. \tag{12}$$

The admittance matrices $[G]_\mu$ of the coaxial-type isolation resistors can be represented according to matrix $[F]$ as

$$[G]_\mu = G_\mu[F] \qquad (\mu = 1, \cdots, M). \tag{13}$$

Therefore,

$$[P_F]^t[G]_\mu[P_F] = \operatorname{diag}[0, G_\mu f_2, \cdots, G_\mu f_n]. \tag{14}$$

To simplify the calculation, each n-wire line is assumed to be n uncoupled transmission lines, then the characteristic admittance matrices $[\eta]_\mu = Y_\mu 1_n$ can be represented as

$$[P_F]^t[\eta]_\mu[P_F] = Y_\mu 1_n \qquad (\mu = 1, \cdots, M). \tag{15}$$

Since all the admittance matrices of the coaxial-type isolation resistors and all the characteristic admittance matrices are transformed into diagonal matrices according to the orthogonal matrix $[P_H]$, the $(1,n)$-port, shown in Fig. 2, can be equivalently transformed into a two-port of an even-mode circuit and the $n-1$ one-ports into the odd-mode circuits. The two-port circuit is represented by a quarter-wave transformer of M sections. As for the $n-1$ one-port circuits, the number of different circuits is $n/2$ (n even) or $(n-1)/2$ (n odd).

By the same method, described in Section II, we can show that the coaxial-type n-way HPD needs $n/2$ (n even) or $(n-1)/2$ (n odd) sections of n-wire lines and coaxial-type isolation resistors for matching at the center frequency.

IV. Three-Way and Four-Way Planar Hybrid Power Dividers

In this section, let's consider the synthesis of three-way and four-way planar HPD's.

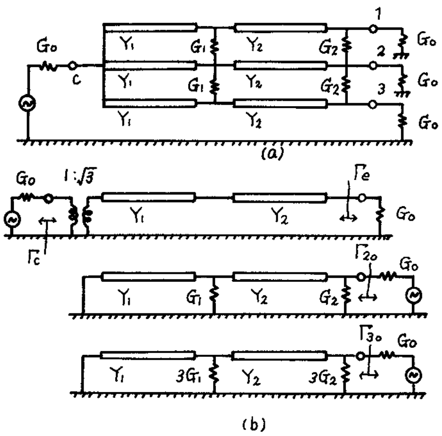

Fig. 3. (a) A three-way planar hybrid power divider. (b) The equivalent circuit representation.

A. Three-Way Planar HPD

As described in Section II, a three-way planar HPD can be obtained by using 2 sections of 3 transmission lines and planar isolation resistors as shown in Fig. 3(a). As for the three-way HPD, the $[H]$ matrix is

$$[H] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}. \tag{16}$$

The eigenvalues of matrix $[H]$ are $h_1 = 0, h_2 = 1$, and $h_3 = 3$, and the orthogonal matrix $[P_H]$ is

$$[P_H] = [\boldsymbol{P}_1 \quad \boldsymbol{P}_2 \quad \boldsymbol{P}_3] = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}. \tag{17}$$

Fig. 3(b) shows the equivalent circuit representation of the three-way HPD. Assuming all the impedances of the input and three output ports be 50 Ω, the two-port circuit is represented by a quarter-wave transformer of 2 sections with a transformation ratio 150 Ω : 50 Ω. If we consider the maximally flat transformer, the characteristic impedances Z_1 and Z_2 are selected as

$$\begin{aligned} Z_1 &= 1/Y_1 = 114.0\ \Omega \\ Z_2 &= 1/Y_2 = 65.80\ \Omega. \end{aligned} \tag{18}$$

In order that the two one-port circuits, respectively, match at the center frequency, the resistances R_1 and R_2 of the planar isolation resistors are selected as

$$\begin{aligned} R_1 &= 1/G_1 = 64.95\ \Omega \\ R_2 &= 1/G_2 = 200\ \Omega. \end{aligned} \tag{19}$$

Let the scattering matrix of the two-port circuit be $[S_e]$

$$[S_e] = \begin{bmatrix} \Gamma_c & T_{ce} \\ T_{ce} & \Gamma_e \end{bmatrix} \tag{20}$$

and let the reflection coefficients of the two one-port circuits be Γ_{20} and Γ_{30}, respectively, and the scattering matrix of the

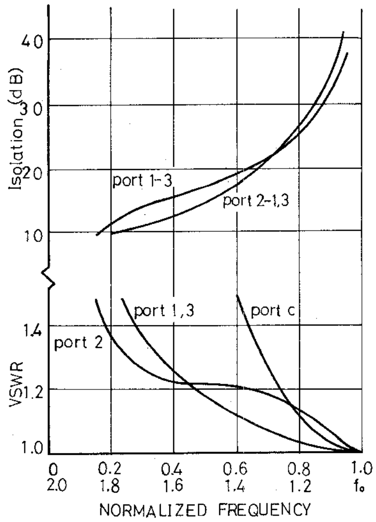

Fig. 4. Theoretical VSWR and isolation responses of the three-way planar HPD.

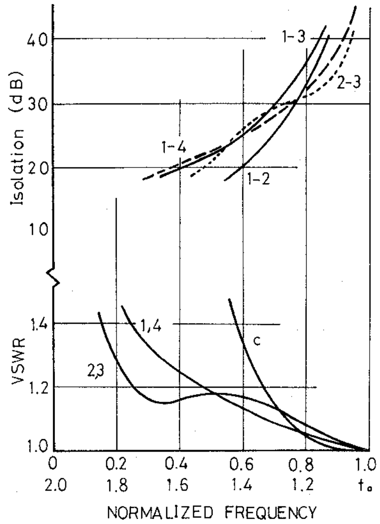

Fig. 6. Theoretical VSWR and isolation responses of the four-way planar HPD.

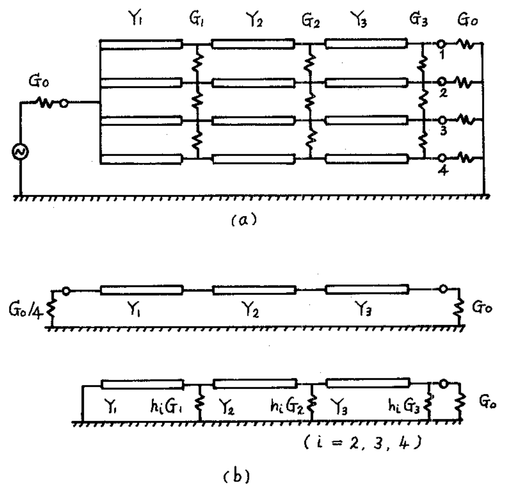

Fig. 5. (a) A four-way planar hybrid power divider. (b) The equivalent circuit representation.

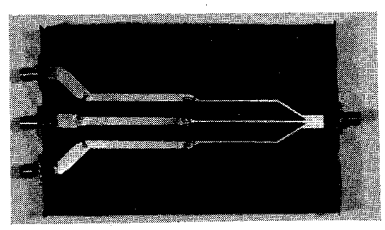

Fig. 7. Photograph of an experimental three-way planar HPD.

three-way planar HPD can be obtained, by the method described in [8], [9], as

$$\begin{bmatrix} \Gamma_c & P_1^t T_{ce} \\ P_1 T_{ce} & \Gamma_e P_1 P_1^t + \Gamma_{20} P_2 P_2^t + \Gamma_{30} P_3 P_3^t \end{bmatrix}. \quad (21)$$

The VSWR and isolation responses are obtained by numerical calculation of the above equation. Fig. 4 shows the characteristics. The isolation characteristics among three output ports of the three-way HPD show more than 20 dB, and the VSWR's of four ports show less than 1.4 in 2 : 1 bandwidth.

B. Four-Way Planar HPD

As described in Section II, a four-way planar HPD can be obtained by using 3 sections of 4 transmission lines and planar isolation resistors as shown in Fig. 5(a). Fig. 5(b) shows the equivalent circuit representation of the four-way HPD. If we consider the maximally flat transformer for the two-port circuit, the characteristic impedances are selected as

$$\begin{aligned} Z_0 &= 1/G_0 = 50\ \Omega \\ Z_1 &= 1/Y_1 = 167.97\ \Omega \\ Z_2 &= 1/Y_2 = 100\ \Omega \\ Z_3 &= 1/Y_3 = 59.54\ \Omega. \end{aligned} \quad (22)$$

In order that three one-port circuits, respectively, match at the center frequency, the resistances of the planar isolation resistors are selected as

$$\begin{aligned} R_1 &= 1/G_1 = 60.46\ \Omega \\ R_2 &= 1/G_2 = 110.27\ \Omega \\ R_3 &= 1/G_3 = 300\ \Omega. \end{aligned} \quad (23)$$

The VSWR and isolation responses of the four-way HPD are shown in Fig. 6. The isolation characteristics among four output ports show more than 24 dB, and the VSWR's of five ports show less than 1.2 in 2 : 1 bandwidth.

C. Experiment of a Three-Way HPD

The photograph of the experimental circuit of a three-way planar HPD described in this section is shown in Fig. 7. The VSWR and isolation responses for the three-way HPD are shown in Fig. 8 with the theoretical curves (solid lines in Fig. 8), and these characteristics are in good agreement with the theoretical analysis.

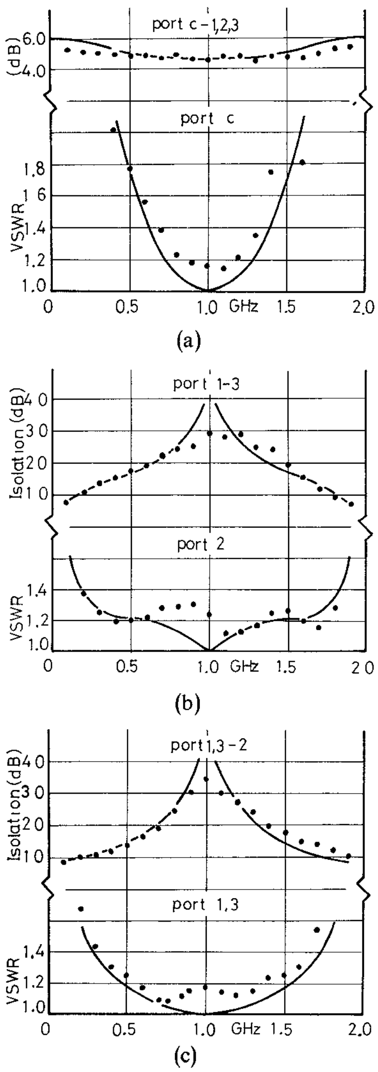

Fig. 8. VSWR and isolation responses of the experimental three-way planar HPD.

V. Conclusion

A new n-way planar HPD and a coaxial-type n-way HPD are proposed, and the synthesis methods of three-way and four-way planar HPD's are shown. The VSWR and isolation characteristics of the experimental three-way planar HPD are also shown, and these characteristics are in good agreement with the theoretical analysis.

We are now trying to design an n-way planar HPD ($n \geq 5$), a coaxial-type n-way HPD, and wide-band n-way HPD's. These circuits will be further applied in the fields of the microwave transmission systems.

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